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Sheet 1

I) Verify the following formulas

$\int_0^\infty \frac{t^{c+1}}{(1+t^2)^2}dt = \frac{c\pi}{4\sin(c\pi/2)}$	$\int_0^\infty \frac{t^{ac-1}}{(1+t^c)^{a+b}}dt = \frac{1}{c}\beta(a,b)$
$\int_0^\infty \frac{t^{a-1}}{(1+ut)^{p+1}}dt = \frac{1}{u^a}\beta(a,p+1-a)$	$\int_0^1 t^{aq-1}(1-t^q)^{b-1}dt = \frac{1}{q}\beta(a,b)$
$\int_{-\infty}^a \frac{(a-t)^{p-1}}{t-b}dt = -\frac{\pi}{\sin p\pi}[b-a]^{p-1}$	$\int_1^\infty \frac{(t-1)^a}{t^b}dt = \beta(a+1,b-a-1)$
$\int_1^\infty \frac{dt}{(a-bt)(t-1)^c} = -\frac{\pi}{b}[\frac{b}{b-a}]^c \csc c\pi$	$\int_0^\infty \frac{t^{a-1}}{t+c}dt = \frac{\pi}{\tan(a\pi)}(-c)^{a-1}$
$\int_0^1 t^{q/p-1}(1-t^q)^{-1/p}dt = \frac{1}{q}\beta(1/p,1-1/p)$	$\int_0^1 \frac{t^{aq-1}}{\sqrt[q]{(1-t^q)}}dt = \frac{1}{q}\beta(a,1-1/q)$
$\int_0^{\pi/2} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}} = \frac{(\Gamma(1/4))^2}{4\sqrt{\pi}}$	$\int_0^1 \frac{t^{3c-m}}{\sqrt[3]{(1-t^3)}}dt = \frac{1}{3}\beta(c+\frac{1-m}{3},\frac{2}{3})$
$\int_0^1 t^{p+q-1}(1-t^q)^{-p/q}dt = \frac{1}{q}\beta(1+p/q,1-p/q)$	$\int_0^\infty \frac{a\,dt}{\sqrt{t}(a^2+t^2)} = \frac{\pi}{\sqrt{2a}}$

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II) Evaluate the following integrals

$\int_{-\pi/4}^{\pi/4} (\sin\theta + \cos\theta)^{1/3} d\theta$	$\int_{0}^{1/2} t^{m-1} \ln(1/2t)\, dt$	$\int_{0}^{\infty} a^{-mx^n} dx$
$\int_{-a}^{a} (a+x)^{m-1}(a-x)^{n-1} dx$	$\int_{0}^{\infty} \frac{x^8(1-x^6)dx}{(1+x)^{24}}$	$\int_{0}^{\infty} x^m e^{-ax^n} dx$
$\int_{0}^{3} \frac{dt}{\sqrt{3t-t^2}}$	$\int_{0}^{1} x^m (\log_a x)^n dx$	$\int_{0}^{\infty} \frac{t^2}{1+t^4} dt$

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Sheet 2

I) Find F(s) of the following functions:

$f(t) = e^{-2.7t}[\cos(9.2t + 3)] + \dfrac{ke^{-k^2/4t}}{\sqrt{4\pi t^3}} + \dfrac{e^{-k^2/4t}}{\sqrt{\pi t}}$

$f(t) = U(t-3)[-e^{5t} + 2 + 3t^2] + 5\sin(5t+8)\,U(t-\pi)$

$f(t) = t\sin 3t\cosh 2t + 4\sin^2 3t$

II) Find inverse Laplace of the following functions:

$F(s) = \dfrac{2 - 3se^{-s} + 4e^{-3s}}{s(s+1)} + \dfrac{s^3 + 9s^2 + 6s + 3}{s^3 + 5s^2 + 4s + 6}$

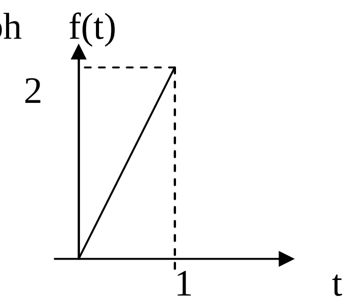
$F(s) = \dfrac{5s^2 + 8s - 5}{s^2(s^2 + 2s + 5)} + \dfrac{9s + 4}{(s+3)^3}$

$F(s) = \dfrac{25}{s^3(s^2 + 4s + 5)} + \dfrac{9s + 4}{(s-3)^2 + 6}$

III) Solve the following differential equations using Laplace:

$y'' + 2y' - 3y = U(t-2)(t-1)$, $y(0) = 1$, $y'(0) = -1$

$y'' + y = f(t)$, $y(\pi/4) = \pi/2$, $y'(\pi/4) = 2-\sqrt{2}$, where f(t) is given by indicated graph



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Sheet 3

1) f(t) = $\begin{cases} \pi^2, & -\pi < t < 0 \\ (t-\pi)^2, & 0 < t < \pi \end{cases}$, with f(t) = f(t + 2 π). Show how to use this Fourier series to compute the sum $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

2) f(x) = $\begin{cases} \pi/2 + x, & -\pi \le x \le 0 \\ \pi/2 - x, & 0 < x \le \pi \end{cases}$. Expand in Fourier series and then deduce the sum $\frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + ...$

3) Let the 2 π periodic function f (x) be specified on the interval (-π, π) by f (x) = x^2.

(a) Determine the coefficients $\{c_k\}$ and $\{d_k\}$ in the Fourier Series expansion f(x) = $\sum_{k=1}^{\infty} c_k \cos kx + \sum_{k=1}^{\infty} d_k \sin kx$, then determine the value of $\sum_{k=1}^{\infty} \frac{1}{k^4}$.

4) Let f(x) be a function of period 2 π such that f(x) = $\begin{cases} x, & 0 < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$,

Sketch a graph of f(x) in the interval −2 π < x < 2 π and find the sum of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$ and $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ...$

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5) Solve the following integral equations

a) $\int_0^\infty f(x)\sin(\alpha x)\,dx = \begin{cases} 1 & 0 \le \alpha < 1 \\ 2 & 1 \le \alpha < 2 \\ 0 & 2 \le \alpha \end{cases}$

b) $\int_0^\infty f(x)\cos(\alpha x)\,dx = \begin{cases} 1-\alpha & 0 < \alpha < 1 \\ 0 & \alpha > 1 \end{cases}$

6) Expand in half range cosine (cosine harmonic) the functions

a) $f(x) = \sin x$, $0<x<\pi$, b) $f(x) = x$, $0<x<1$, c) $f(x) = e^x$, $0<x<\pi$

d) $f(x) = x(\pi - x)$, $0 < x < \pi$, then deduce the sum $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

7) Expand in half range sine (sine harmonic) the following functions

a) $f(x) = x^2$ $0<x<2$, b) $f(x) = \cos x$ $0<x<\pi$,

c) $f(x) = x(\pi - x)$, $0 < x < \pi$, then deduce the sum $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}$

8) Expand Problems in 6 and 7 in odd harmonic and in Even harmonic

9) Expand in complex Fourier

a) $f(x) = e^x$, $-\pi<x<\pi$, b) $f(x) = x$, $-1<x<1$

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10) Expand the function f(x) = x , 0 < x < T in

a) Fourier cosine
b) Odd harmonic
c) Even Cosine harmonic

11) Find Fourier transforms of the following functions

a) $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$, then evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos(x/2)\, dx$

b) $f(x) = \begin{cases} 1 & 0 < x < a \\ -1 & -a < x < 0 \\ 0 & |x| > a \end{cases}$

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Sheet 4

1) **Evaluate the following integrals**

1- $\int_c f(z)\,dz$, where c is the curve $y = x^3$ from -1-i to 1+i

$$f(z)=\begin{cases}1 & y<0\\ 4y & y\geq 0\end{cases}$$

2- $\int_c (z+\bar{z})dz$, where c is the path from z = 0 to z = 1+2i consisting of line segment from z = 0 to z = 1 together with the line segment from z = 1 to z = 1+2i.

3- $\int_{(-1,0)}^{(2,3)} (3x^2+3y)\,dx + (3x-4y)\,dy$, along the path $y^2 - 2x^2y+x^4 -1 = 0$

4- Find the harmonic conjugate for the following:

a) $u(x,y) = x^2 - y^2 + y$ b) $v(x,y) = 2xy + 3y$

5- If $f(z) = e^x\cos(ay) + i\, e^x\sin(y-b)$ is differentiable at every point, then find a and b.

6- $\int_c \frac{\cos(z)}{z^2-6z+5}dz$ where c is the circle $|z| = 4$.

7- $\int_c \frac{z^3+5z+7}{(z-i)^2}dz$ c: $|z-2| + |z+2| = 6$

9- $\int_c \frac{dz}{(z-4)^3}$ c: $x = 3\cos t$, $y = 2\sin t$

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Sheet 5

1- Use Picard's method up to third approximation the following differential equation $y'' = x^3 (y' + y)$, $y(0) = 1$, $y'(0) = ½$

2- Find y(0.3) for the D.E. $y' = 3x + y^2$, $y(0) = 1$ using Euler method, h=0.1.

3- Solve the following differential equation $y' = x^2 - y^2$, $y(0) = 0$, using Taylor method

4- (1,3) , (5,-7) , (-13,4) , (2,47) , (-6,15)

Find Lagrange interpolating polynomial that fit the above data.

5-Construct Newton interpolation polynomial satisfy the following data

x	3.5	3.55	3.6	3.65	3.7
y	33.115	34.413	36.598	38.475	40.447

6- Solve the linear system by Gauss-Seidel Method

a) x +4y+z=2, 4x+y+z = 5, x+y+4z =3

b) x+y+5z=3, 6x-2y+2z = 5, 3x+9y+4z =11

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